COMPARATIVE ANALYSIS OF DIFFERENT METHODS OF CALCULATING PRESSURE INSIDE THE BARREL IN POST-MUZZLE PERIOD OF A SHOT

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Abstract: *Results of calculating pressure inside the barrel in post-muzzle stage of a shot were shown. In conducted investigations, obtained pressure values were used to assess influence of different methods of describing post-muzzle stage of a shot on behavior of recoil operated firearm operation. Results of performed analysis shown that differences between compared methods are negligible when comparing obtained values of pressure and recoil velocities, which were calculated with the use of mathematical model of recoil operated firearm, based on thermodynamic model of internal ballistics.*

Keywords: *internal ballistics, post-muzzle stage of a shot, firearm operation, recoil.*

1. Introduction

Post-muzzle stage of a shot is part of a gun discharge process which takes place after the projectile leaves the barrel bore. Said stage lasts until propellant gas pressure drops to the level of atmospheric pressure. Post-muzzle stage of a shot is important especially in recoil-operated weapons, where force generated by propellant gas pressure acts on face of a bolt locked with barrel, causing increase in recoil velocity of recoiling assembly (compromised of said bolt and barrel) (fig. 1).

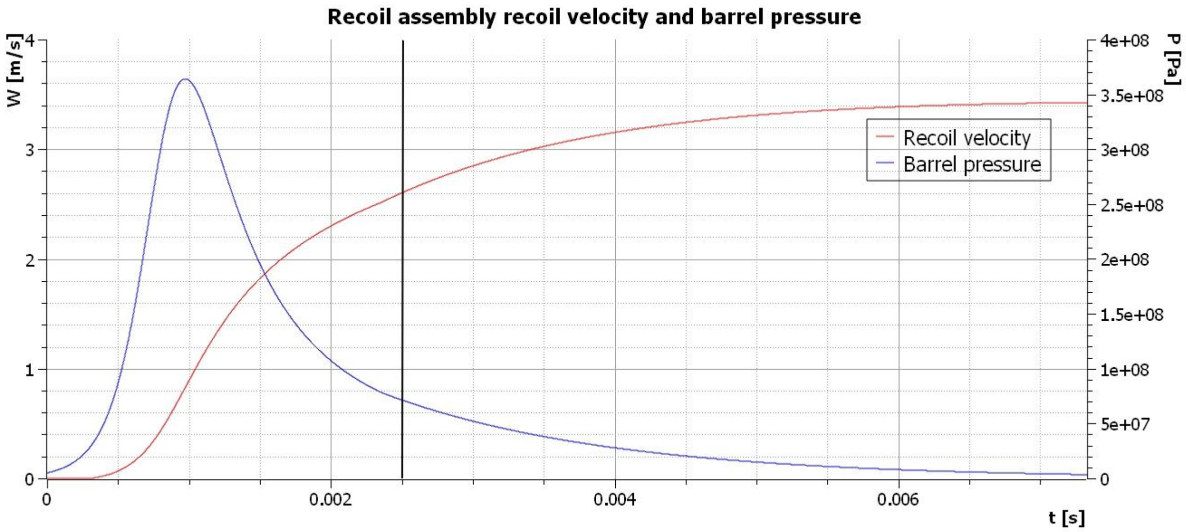

Fig. 1. Graph comparing the propellant gas pressure inside the barrel and recoil velocity of recoiling assembly, black vertical line marks moment of bullet leaving barrel (based on [1])

Most common method of calculating post-muzzle pressure is based on Brawin’s empirical formula, which describes exponential drop of pressure inside the barrel [2]. Due to ease of use of Brawin’s method, it is treated as a reference method in presented work. Input parameters for the thermodynamic model of internal ballistic and physical and mathematical model of recoil operated firearm operation were presented in [1].

2. Methods of calculating pressure in post-muzzle stage of a shot

2.1. Brawin's formula

Propellant gases pressure evolution during the post-muzzle stage of a shot is described with Brawin's empirical formula (1) [2]:

$$p = p_w \cdot e^{-\frac{t_{op}}{b}}. \qquad (1)$$

where:
p_w - gas pressure at the moment of leaving barrel by the projectile,
t_{op} - time in post-muzzle stage of a shot,
b - constant for Brawin's formula.

Constant value b is calculated by the formula (2) [2]:

$$b = \frac{\beta - 0{,}5 \cdot \omega}{s \cdot p_w - p_{pw}} \cdot v_w \qquad (2)$$

where:
β - coefficient of post-muzzle gas effect,
ω - mass of propellant,
s - barrel bore cross-sectional surface area,
p_{pw} - gas pressure at the end of post-muzzle stage (atmospheric pressure),
v_w - muzzle velocity of the projectile.

Value of the coefficient of post-muzzle gas effect β, present in relationship (2), is expressed by the empirical formula (3) [2]:

$$\beta = 1{,}5 + \frac{6{,}45}{\frac{p_m}{p_w} \cdot \frac{l_w}{d} \cdot 0{,}23}. \qquad (3)$$

where:
p_m - maximum gas pressure inside the barrel,
l_w - overall projectile travel inside the barrel,
d - calibre.

2.2. Outflow of gases from the barrel

In order to determine validity of application of the relationship describing outflow of propellant gases from the barrel to the environment, an analysis of the relationship between flow velocity of the gas-powder mixture and wave velocity in the gas-powder mixture after the projectile exit was carried out. Flow velocity of the mixture was determined by carrying out calculations with internal ballistics thermodynamic model for an elongated barrel and using the relationship (4):

$$v_m = v \frac{l_w}{l} \qquad (4)$$

where:
v_m - velocity of the gas-powder mixture,
v - current bullet velocity,
l_w - projectile travel at the moment of leaving the muzzle (for a standard barrel length),
l - current projectile travel (for an elongated barrel).

Velocity of the wave in the gas-powder mixture was determined using the equation (5) [3]:

$$c = \sqrt{\frac{kp}{\rho^2 \frac{1}{\rho} - \frac{1-\psi}{\rho_p} - \alpha\psi}} \qquad (5)$$

where:
k - propellant gas adiabatic exponent,
p - pressure in the barrel bore,
ρ - density of the gas-powder mixture,
ρ_p - propellant density,
ψ - relative volume of the burnt propellant,
α - propellant gases co-volume.

Density of the gas-powder mixture was determined from the formula (6):

$$\rho = \frac{m_p}{W_0 + ls} \tag{6}$$

where:
m_p - propellant charge mass,
W_0 - volume of the combustion chamber,
l - projectile travel in the barrel bore,
s - barrel bore cross-sectional area.

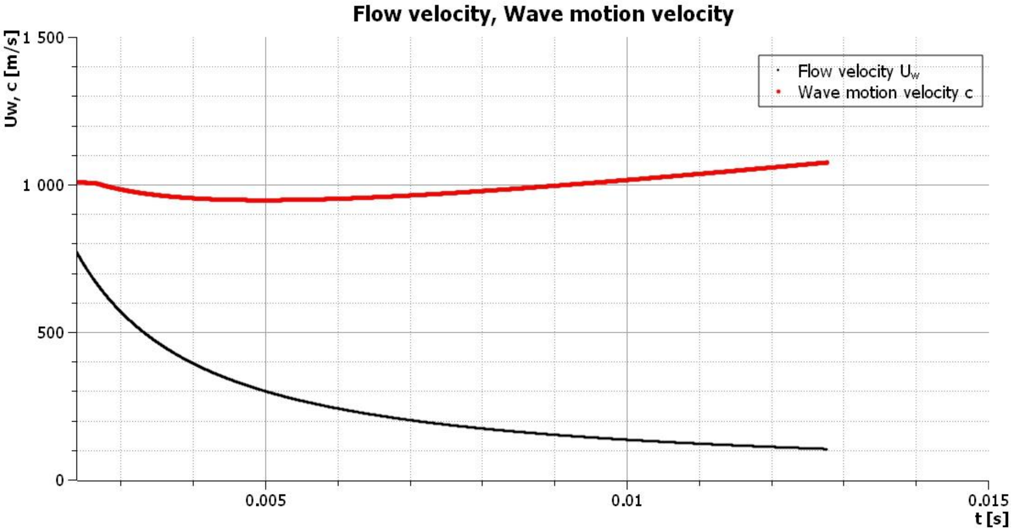

Fig. 2. Graph comparing flow velocity and velocity of the wave motion after the projectile exit

Due to subsonic velocity of the propellant gas flow after the projectile exits the barrel, it is possible to use dependencies describing outflow of propellant gases from the barrel to the environment.

2.2.1. Propellant burning after leaving barrel by projectile

Equation of energy balance in the barrel taking into account combustion of propellant charge and outflow of gases from the barrel bore to the environment (7) [4]:

$$\frac{dRT}{dt} = \frac{\frac{d\psi}{dt}\left(\theta q_s - RT\right) - \theta RT \frac{d\gamma}{dt}}{\psi - \gamma} \tag{7}$$

where:
R – gas constant,
T – gas temperature,
t – time,
θ - function of the adiabatic exponent of propellant gas ($\theta = k-1$),
q_s - heat of propellant combustion,
γ - relative volume of the propellant gas that flowed from the barrel to the environment.

Equation of state of propellant gases in the barrel (8) [4]:

$$p = \frac{\omega\left(\psi - \gamma\right)RT}{W_0 + sl - \frac{\omega}{\rho_p}\left(1 - \psi\right) - \alpha\omega(\psi - \gamma)} \tag{8}$$

Equation of relative volume rate of propellant charge combustion (9) [4]:

$$\frac{d\psi}{dt} = \frac{S_1}{\Lambda_1}\sqrt{1 + 4\frac{\lambda_1}{\chi_1}\psi} \cdot u_1 p \qquad (9)$$

where:
S_1 - propellant grain initial surface area,
Λ_1 - propellant grain initial volume,
u_1 - coefficient of linear burning rate.

Equation of relative mass velocity of propellant gas outflow from the barrel to the environment (10) [4]:

$$\frac{d\gamma}{dt} = \frac{\xi_w}{\omega}\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}\sqrt{\frac{2k}{k+1}} \cdot \frac{p}{\sqrt{RT}} \qquad (10)$$

where:
ξ_w – loss coefficient.

2.2.2. Outflow of gases from the barrel after complete propellant burn

Equation of energy balance in the barrel after complete combustion of the propellant charge (ψ = 1), taking into account outflow of propellant gases from the barrel bore to the environment (11) [4]:

$$\frac{dRT}{dt} = \frac{-\theta RT\frac{d\gamma}{dt}}{1-\gamma} \qquad (11)$$

Equation of state of propellant gases in the barrel (12) [4]:

$$p = \frac{\omega(1-\gamma)RT}{W_0+sl-\alpha\omega(1-\gamma)} \qquad (12)$$

Equation of relative mass velocity of propellant gas outflow from the barrel to the environment stays the same as in the stage of propellant burning (10).

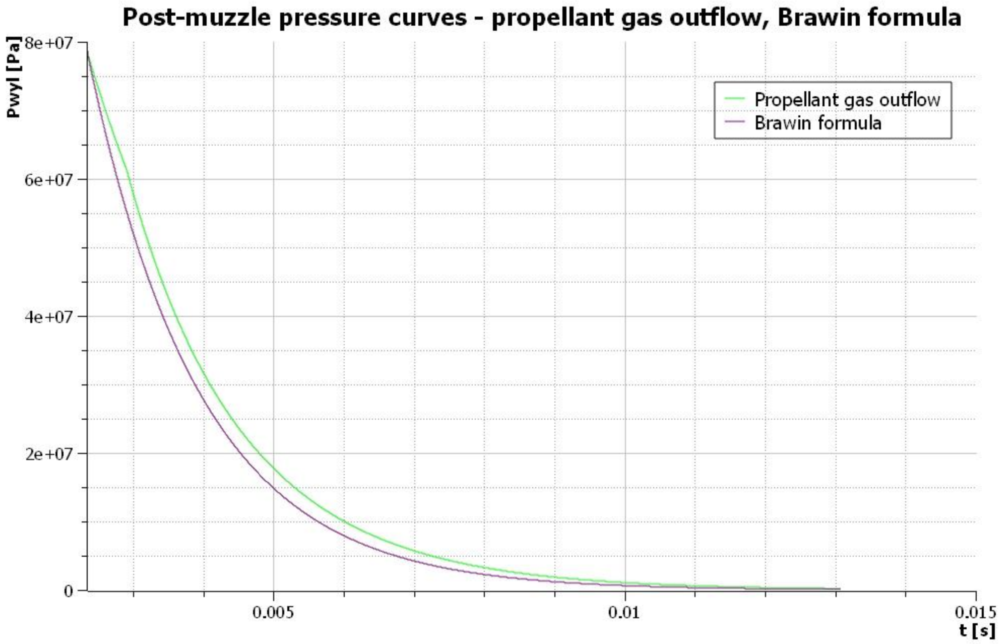

Fig. 3. Comparison of pressure curves in post-muzzle period determined using Brawin's formula and equations of outflow of gases from the barrel bore

Comparison of pressure curves obtained from both methods visible in the graph (Fig. 3), shows significant deviation of the curves in the initial part of the post-muzzle period which is caused by the afterburning of the propellant charge. Calculations carried out with the thermodynamic model of internal ballistics show that in analyzed case, about 93.5% of the propellant charge was burnt when the projectile exited the barrel.

2.3.Propagation of rarefaction wave from muzzle to the bottom of the barrel bore

Third method of mathematical description of the post-muzzle period of a shot is based on calculations for a fictitious elongation of the barrel. It is based on the fact that the pressure acting on the breech for the elongated barrel changes in the same way as for the real barrel until the rarefaction wave (formed when the projectile leaves barrel muzzle) traveling through the barrel bore reaches the bottom of the combustion chamber. Rarefaction wave is treated as an ‘information’ about opening of the barrel bore – when the front of the wave reaches bottom of the combustion chamber, pressure in the barrel bore starts to drop rapidly. Concept of rarefaction wave propagation in the barrel bore was also used in the Rarefaction Wave Gun (RAVEN) project, where the phenomena was used to decrease recoil of large caliber guns by opening barrel bore in the breech area, allowing to vent propellant gases from the barrel. Venting gases from the barrel was synchronized with rarefaction wave position, not allowing to reach base of the projectile by the wave – rarefaction wave reached barrel muzzle after the projectile left the barrel bore, thus preventing pressure to drop while the projectile was still accelerating inside the barrel [5].

Determination of the characteristic time of wave processes, justifying the inclusion of said processes in modeling, begins with comparison of the characteristic times of wave motion and the motion of the projectile (fig. 4).

Curve of characteristic time of wave transport process T_f in relation to the duration of the firing process was determined by referring wave velocity to sum of lengths of projectile travel in the barrel and the combustion chamber (13):

$$T_f = \frac{l_k + l}{c} \tag{13}$$

where:
l_k - length of the combustion chamber.

Rarefaction wave velocity was calculated by using formulas (5) and (6).

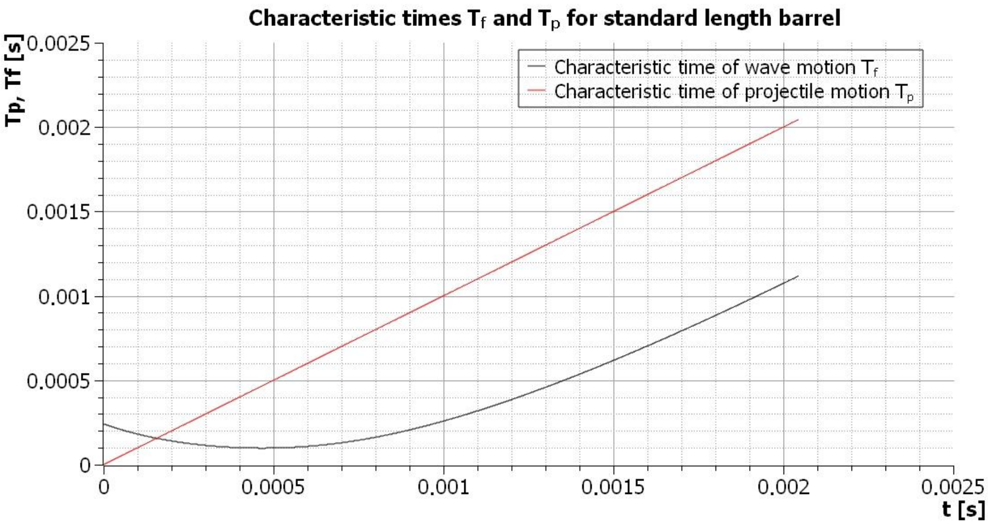

Fig. 4. Comparison of the characteristic times of wave motion and projectile motion for a standard length barrel (the length used in the actual layout)

Based on the graph showing the relation between the characteristic times of projectile motion and wave transport processes (fig. 4), it can be concluded that it is reasonable to take wave processes into account in the modeled phenomenon due to the same order of magnitude of both times.

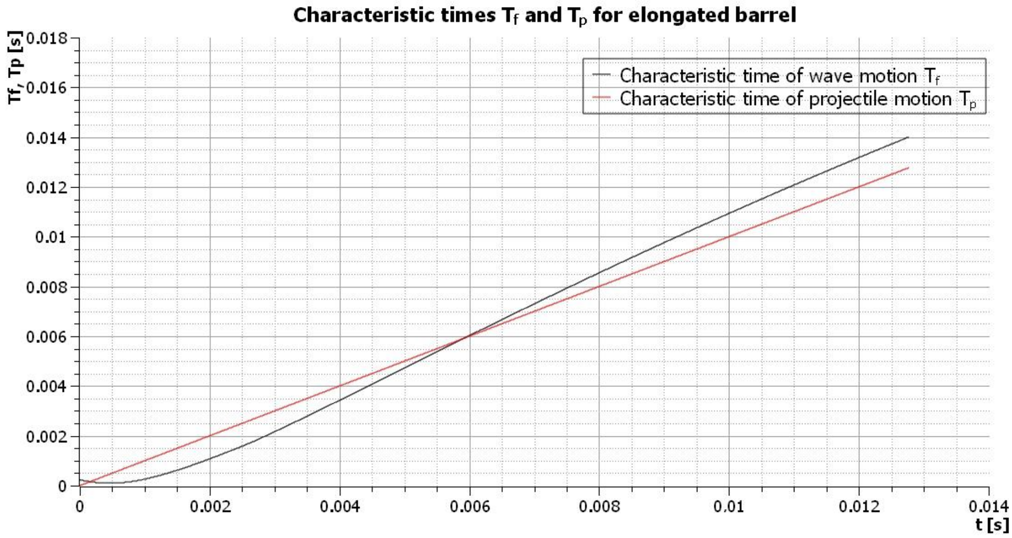

Fig. 5. Comparison of characteristic times of wave motion and bullet motion for an extended barrel

In case of comparing times T_p and T_f for an extended barrel (analysis was carried out for duration of the phenomenon equal to duration of the pressure applied to the breech assembly determined using the Brawin’s formula) (fig. 5), in the entire duration of the phenomenon, both characteristic times are characterized by the same order of magnitude. Increasing the value of time T_f in case of an elongated barrel is caused by increase in volume, and thus a decrease in density of the medium (gas-powder mixture) in which wave transport processes take place.

Taking into account wave transport processes in operation of the modeled system was carried out by fictitious extension of the barrel bore until the rarefaction wave moving deeper into the barrel reaches the bottom of the combustion chamber (fig. 6). Position of the rarefaction wave front was determined by relating distance traveled by it to sum of length of the barrel bore and the combustion chamber. The path of the wave front was determined by integrating wave velocity over time, being the difference between the flow velocity and the velocity of sound in the gas-powder mixture (due to opposite directions of the mixture and the wave front).

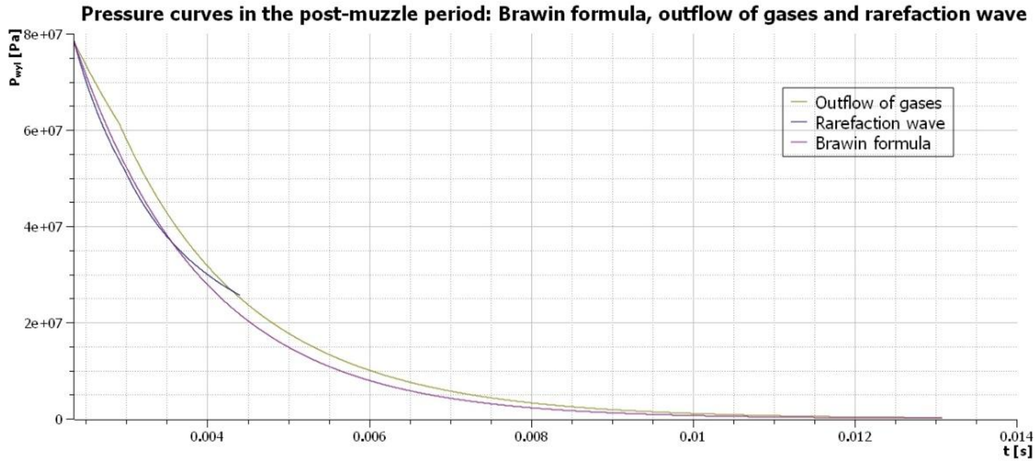

Fig. 6. Comparison of pressure curves in the post-muzzle period, determined using:
the Brawin’s formula, the relationships describing the outflow of gases from the barrel tube to the environment and a fictitious extension of the barrel bore during the transition of the rarefaction wave to the bottom of the combustion chamber

Comparison of calculation results for the elongated barrel, the Brawin’s formula and the equations of gas outflow to the environment shows that in analyzed case, almost immediately when the rarefaction wave reaches bottom of the combustion chamber, pressure curve for the long barrel intersects with curve determined from the gas flow equations - curves intersect occurs at time t = 0.0043 s, while the rarefaction wave reaches the bottom of the chamber at t = 0.0044 s. Therefore, in order to determine pressure curve until the end of the post-muzzle period, it was decided to use a simplification in form of a direct transition from the curve for elongated barrel to the curve for the outflow of gases from the moment of their intersection (fig. 7).

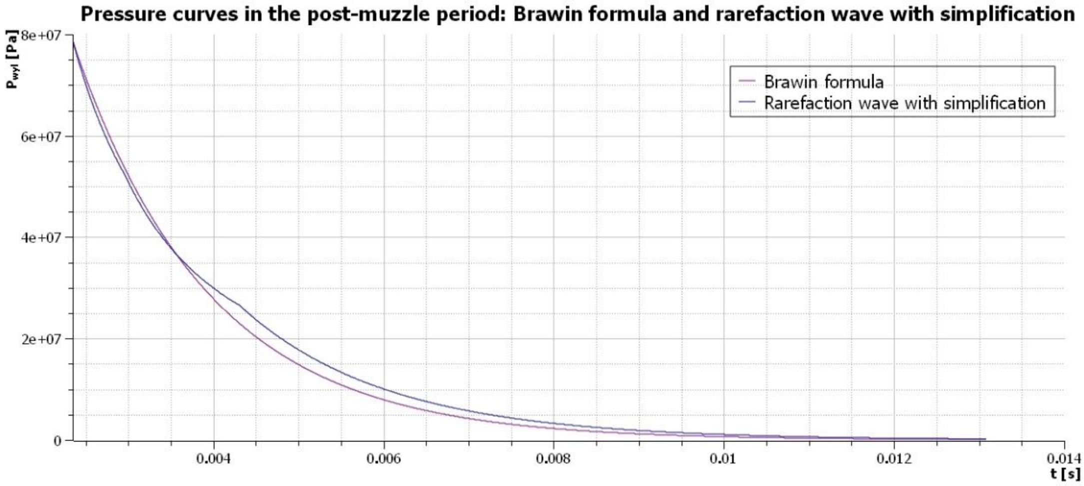

Fig. 7. Comparison of pressure curves in post-muzzle period, determined using the Brawin's formula and a fictitious elongation of the barrel during the transition of the rarefaction wave, which turns into the outflow of gases to the environment

3. Comparison of methods

The diagram (fig. 8) shows comparison of pressure courses in the barrel bore during post-muzzle period for the three methods used. On the basis of the summary, it is possible to notice similar course of all curves in the entire post-muzzle period. The lowest pressure values in almost entire run were obtained using the Brawin's formula. In the initial part of the post-muzzle period, value of the pressure obtained for elongated barrel is lower by about 3%, at the time t = 0.00355 s it intersects with the curve obtained for the Brawin's formula and its slope decreases. Highest pressure values in the entire period were obtained for method based on the outflow of gases from the barrel to the environment. The list of selected parameters for the entire pressure courses and for post-muzzle period is presented in Table 1. Calculations were made using mathematical model of short recoil operated firearm with accelerator described in [1].

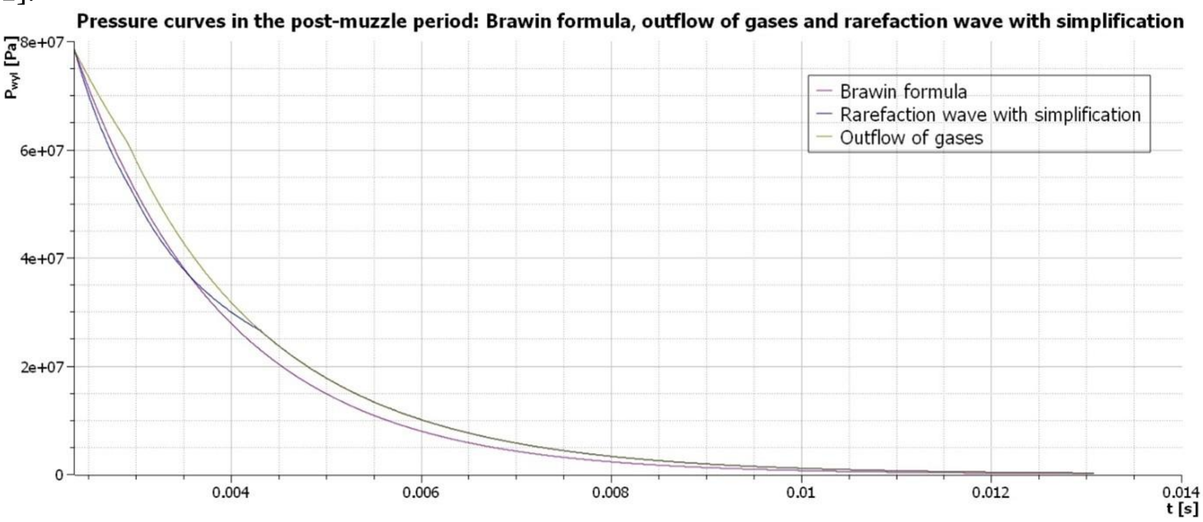

Fig. 8. Comparison of pressure curves in the post-muzzle period obtained with use of three methods

Tab. 1. Selected parameters for the entire pressure courses and for post-muzzle period obtained with different methods:

Brawin's formula		Outflow of gases		Rarefaction wave	
t_k [s]	0.0115	t_k [s]	0.01118	t_k [s]	0.01134
p_k [MPa]	0.258	p_k [MPa]	0.558	p_k [MPa]	0.511
max W_z [m/s]	8.531	max W_z [m/s]	8.952	max W_z [m/s]	8.76
min W_l [m/s]	1.182	min W_l [m/s]	1.239	min W_l [m/s]	1.22
W_r [m/s]	3.427	W_r [m/s]	3.55	W_r [m/s]	3.484
t_r [s]	0.007	t_r [s]	0.007	t_r [s]	0.007
I_p [MPa*s]	0.124	I_p [MPa*s]	0.142	I_p [MPa*s]	0.134

The t_k parameter presented in Table 1 refers to the accelerator operation termination time determined with the use of post-muzzle period calculation methods discussed in the paper. The p_k parameter corresponds to theoretical value of pressure in the barrel bore at t_k. In calculations of the automatics operation, a simplification was used in form of pressure acting on the bottom of the combustion chamber only until the accelerator operation was finished. Simplification was adopted due to relatively small protrusion of the cartridge case from the cartridge chamber during the accelerator operation (at the end of acceleration process, cartridge case protrudes from the chamber to a distance of about 22 mm, while length of the cartridge case in analyzed system is 99 mm, of which length of the chamber is ~ 93.8 mm). In addition, the table also shows: recoil time of the bolt assembly (t_{odrz}), maximum recoil velocity of the bolt (W_z), minimum recoil velocity of the barrel (W_l) (velocity of the barrel after acceleration of the bolt - at the moment of its stopping), recoil assembly recoil velocity at the moment of separation of the bolt and barrel (W_{rozdz}) and corresponding time span. The table also includes values of pressure impulse in the post-muzzle period I_p expressed in MPa * s.

Based on the analysis of the parameters listed in Table 1, there is a relatively small difference between values of parameters determined by the use of various methods of describing the post-muzzle period. Despite maximum difference between the pressure impulses reaching 14.3% (between the outflow of gases and the Brawin’s formula), differences between the maximum bolt recoil velocities, barrel recoil velocities and the velocities at the moment of separation of the parts do not exceed 5% when compared to the velocities determined using the Brawin’s formula. In case of times t_{rozdz} and t_{konc}, the differences do not exceed 0. 3 ms and 0. 1 ms respectively, recoil time t_{odrz} of the bolt is in range of 26.5 ms - 27.6 ms (the difference is 1 ms). Analysis of the pressures p_{konc} corresponding to moments t_{konc} of the end of acceleration of the bolt allows for the validation of the introduced simplification concerning taking into account the pressure only until the end of the accelerator operation. At the end of its operation pressure in the barrel bore is at a relatively low level, in the range of 0.258 MPa - 0.558 MPa (atmospheric pressure is about 0.1 MPa). Compiled curves of the pressures and the recoil velocities of the barrel and breech from the moment the projectile exits the barrel are shown in Fig. 9.

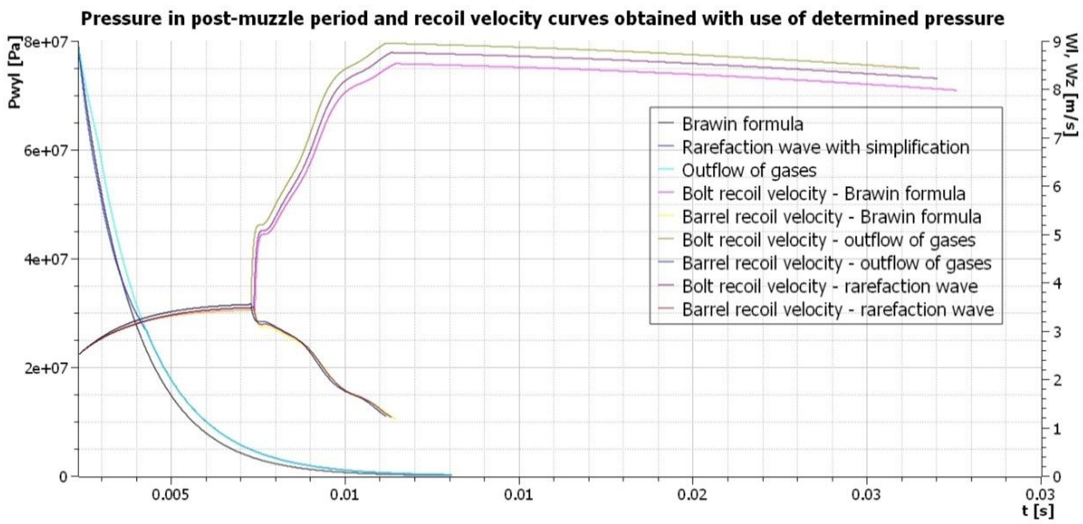

Fig. 9. Comparison of curves of the recoil velocity of the recoiling assembly elements, determined by taking into account obtained pressure values in the post-muzzle period

4. Conclusions

1. Taking into account the phenomena of wave motion is justified in the case of modeling the post-muzzle period of a gunshot. Taking into account passage of the rarefaction wave through the barrel bore and the combustion chamber combined with the outflow of gases from the barrel to the environment gives results on the level of other analyzed methods.
2. Due to length of the barrel of the analyzed system, afterburning of the powder charge (in outflow of the gases from the barrel method) results in obtaining highest value of the post-muzzle pressure impulse in relation to other methods - higher by 14.3% compared to the values of pressure obtained using the Brawin's formula. Despite the differences in value of the pressure impulse, relatively large mass of the elements of the recoiling assembly and presence of the return springs result in a slight increase in the recoil velocity of the elements - the greatest difference is 4.9% (gas flow from the barrel to the environment in relation to the Brawin's formula).

References

[1] Szupieńko D., Woźniak R. 2021. „ Preliminary Physical and Mathematical Model of the Recoil Operated Firearm within the Bolt Recoil Period". Problems of mechatronics: armament, aviation, safety engineering, vol. 12 (1).

[2] Serebriakow M. 1955. Balistyka Wewnętrzna. Warszawa: Wydawnictwo MON.

[3] Landau Lew D., Lifszyc Jewgienij M. 2009. Fizyka teoretyczna – hydrodynamika Warszawa: Wydawnictwo Naukowe PWN.

[4] Leśnik G., Surma Z., Torecki S., Woźniak R. 2009. „Termodynamiczny model działania broni z odprowadzeniem gazów prochowych w okresie napędzania suwadła". Biuletyn WAT vol. LVIII (3).

[5] Kathe E., 2001, ,,Rarefaction Wave Gun Propulsion", Rensselaer Polytechnic Institute.